

Mach's Principle and a Variable Speed of Light

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Why do you believe in GM/r^2 ?

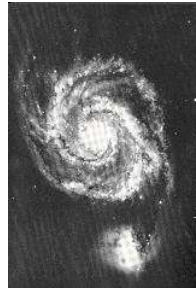
Test in the range 10^4 m,
 10^6 kg $\rightarrow F=mg$



Test in the range 10^{12} m,
 10^{30} kg $\rightarrow F=GM/r^2$



No Test available for
 10^{20} m, 10^{41} kg !



No Test available for
 10^{26} m, 10^{53} kg !



Problems:

- SNIa \rightarrow Dark energy
- Rotation Curves etc.
 \rightarrow Dark matter
- Pioneer anomalous acceleration
 \rightarrow *Dark Substance?*
- Discrepant G measurements
 \rightarrow *Dark Physics ?*

`In fact, there is little or no direct evidence that conventional theories of gravity are correct on scales much larger than a parsec or so. Newtonian gravity works extremely well on scales of 10^{14} cm (the solar system). (...) It is principally the elegance of general relativity and its success in solar system tests that lead us to the bold extrapolation that the gravitational interaction has the form GM/r^2 on the scales 10^{21} - 10^{26} cm...'

(J. Binney and S. Tremaine, Galactic Dynamics 1994)

Mach's principle

- The origin of inertia is that a mass is accelerated with respect to all other masses in the universe.
- The reason for gravitation is the distribution of masses in the universe.

- Dirac's great number hypothesis (1938):

$$G \approx c^2 \frac{M_U}{R_U}$$

- Why is the universe flat ($\Omega=1$) ?
- Has the universe the critical density ?

Sciama's version of Mach's principle

$$\frac{c^2}{G} = \sum_i \frac{m_i}{r_i}$$

$$\Phi = -G \sum_i \frac{m_i}{r_i} = -c^2$$

- Quantitative hypothesis about the nature of the gravitational constant G (MNRAS 113, p.35, 1953)
- Applied to the gravitational potential yields $\Phi = \text{const.}$, if $c = \text{const.}$... Is the gravitational potential related to c ?

Einstein, 1907, 1911:

“The constancy of the velocity of light can be maintained only insofar as one restricts ... to ... regions with constant gravitational potential...”

“... the velocity of light in the gravitational field is a function of the place, [thus] we may easily infer, by means of Huyghens's principle, that light-rays propagated across a gravitational field undergo deflexion”

$c = \lambda f = 299792458$ m/s by definition, but...
 you don't notice a change in c if length
 scales λ and time scales $1/f$ change:

$$\frac{dc}{c} = \frac{df}{f} + \frac{d\lambda}{\lambda} \quad \Downarrow\Downarrow = \Downarrow + \Downarrow$$

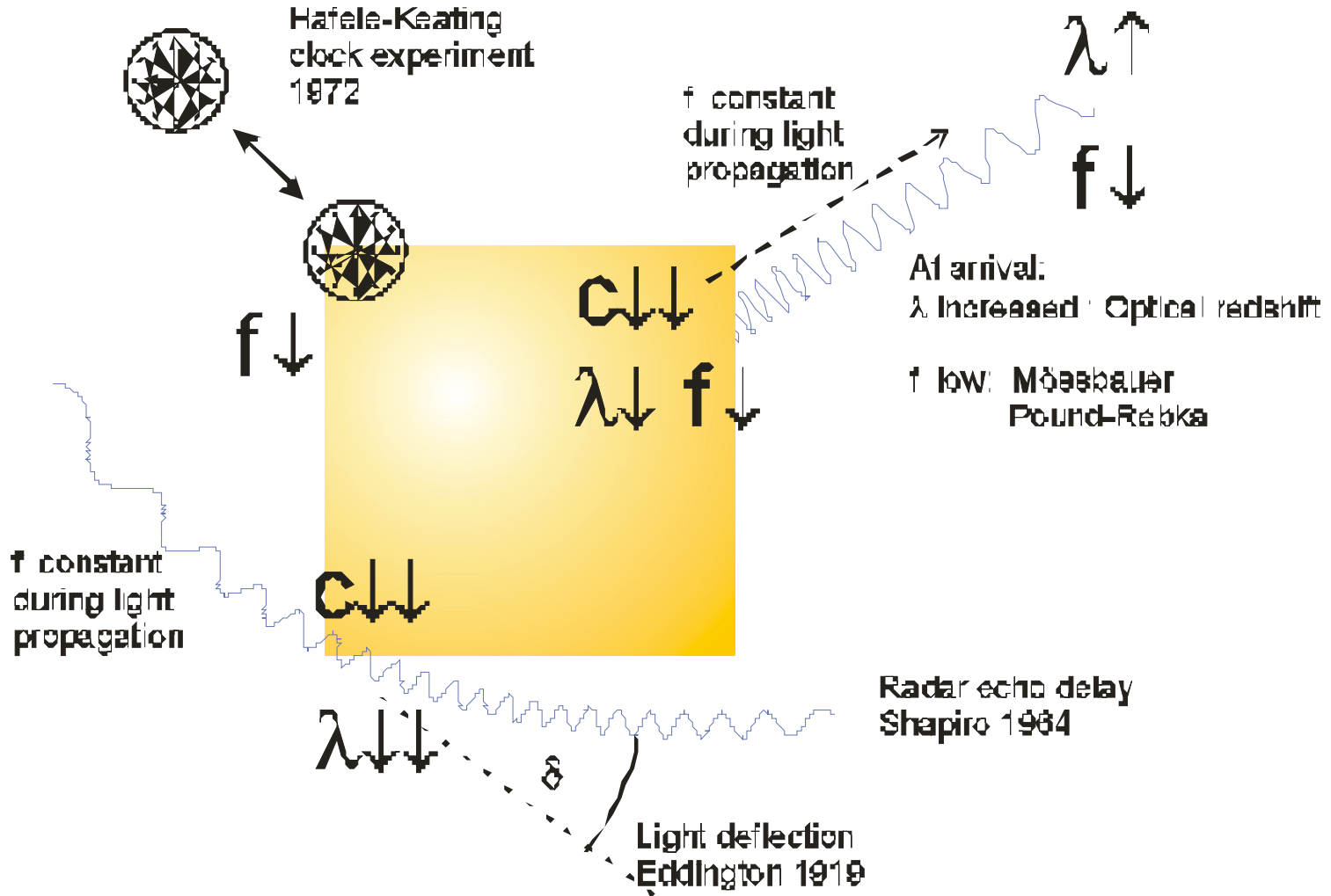
• Hypothesis 1:

$$\frac{dc}{c} = \frac{2GM}{rc^2} \frac{dr}{r} \Downarrow\Downarrow$$

$$\frac{df}{f} = \frac{GM}{rc^2} \frac{dr}{r} \Downarrow$$

$$\frac{d\lambda}{\lambda} = \frac{GM}{rc^2} \frac{dr}{r} \Downarrow$$

- $c \Downarrow\Downarrow$, $f \Downarrow$, and $\lambda \Downarrow$ are lowered in the gravitational field
- Hypothesis 2: $f \rightarrow (f = \text{const.})$ during photon propagation
- 1+2 explains time+length tests of GRT

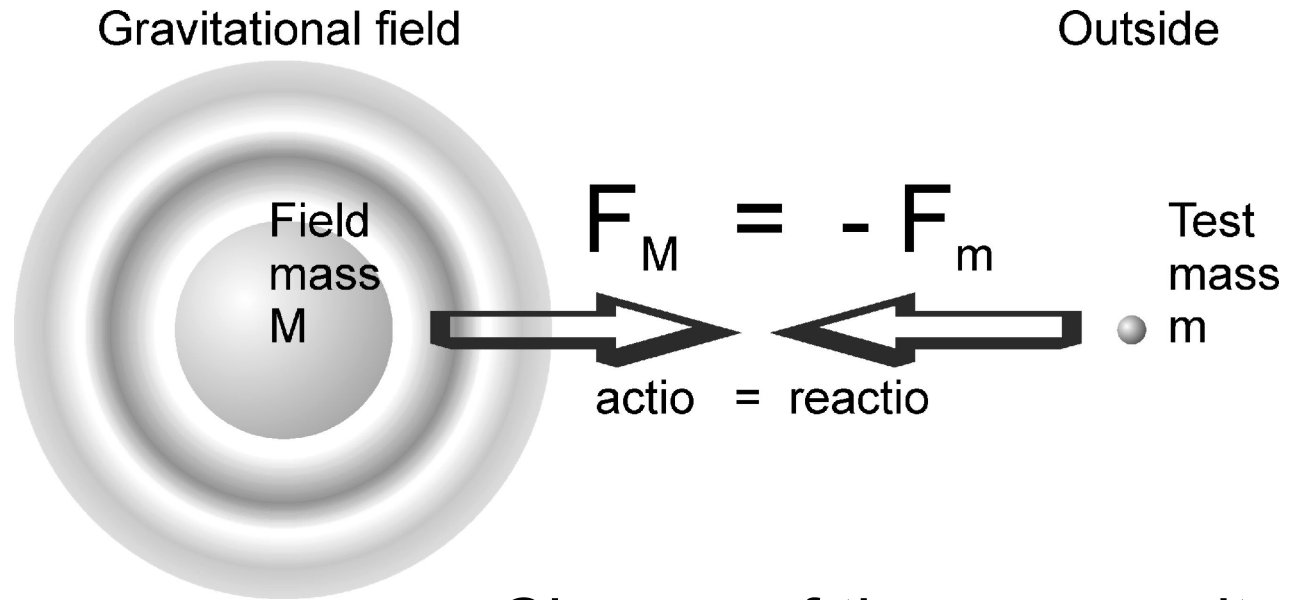


The velocity of light is lower in the gravitational field:

$c \downarrow \downarrow$ $f \downarrow$ $\lambda \downarrow$

Every measurement involving time and length scales changes:

Velocity v (m/s) $\downarrow \downarrow$ acceleration a (m/s²) $\downarrow \downarrow \downarrow$
 $F = m a \Rightarrow$ mass measurement changes: $m \uparrow \uparrow \uparrow$



Change of the mass unit in the gravitational field:

$$\frac{dm}{m} = \frac{3GM}{rc^2} \frac{dr}{r}$$

The total energy of a test particle $E=mc^2$ does not change while moving in a gravitational field

$$dE = d(mc^2) = d\left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} c^2\right) =$$

$$\frac{c^2 dm_0}{\sqrt{1-\frac{v^2}{c^2}}} + d\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)m_0 c^2 + \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} d(c^2)$$

Note: every term is proportional to m_0 !

$$\frac{3GMm_0}{r^2} dr + \frac{1}{2} m_0 v^2 - \frac{4GMm_0}{r^2} dr$$

$$E_{\text{kin}} + E_{\text{pot}} = 0, \text{ as it should be...}$$

E_{kin} is proportional to the inertial mass,

E_{pot} is proportional to the gravitating mass

The equivalence principle follows !

Spatial dependence of c includes Newton's law of gravitation:

Local actual c $c^2(m_i, r_i) = \frac{c_0^2}{\ln \sum \frac{m_i r_p}{r_i m_p}} =: \frac{c_0^2}{\ln \Sigma}$

sum taken over all masses i
 r_i distance to m_i

r_p proton radius
 m_p proton mass

$$\vec{a} = -\frac{1}{4} \nabla c^2 = \frac{-c_0^2}{4(\ln \Sigma)^2} \frac{\sum \frac{m_i \vec{r}}{r_i^3}}{\sum \frac{m_i}{r_i}} \quad G = \frac{c_0^2}{4(\ln \Sigma)^2} \frac{1}{\sum \frac{m_i}{r_i}}$$

Suffices visible matter ?

$$G = \frac{c_0^2}{4(\ln \Sigma)^2} \frac{1}{\sum \frac{m_i}{r_i}} = \frac{c^2}{4 \ln \Sigma} \frac{1}{\sum \frac{m_i}{r_i}}$$

For an homogeneous universe, holds

$$\sum \frac{m_i}{r_i} \approx \frac{3}{2} \frac{M_u}{R_u}$$

Thus,
$$M_u \approx \frac{c^2 R_u}{6G \ln \Sigma}$$

$\approx 3.5 \cdot 10^{50}$ kg, corresponding to $\Omega_{\text{vis}} = 0.004$, is in approximate agreement with observations

with

$$r_p = 1.2 \cdot 10^{-15} \text{ m}$$

$$m_p = 1.67 \cdot 10^{-27} \text{ kg}$$

$$\Sigma \approx 10^{37} \quad \ln \Sigma \approx 85$$

$$R_p = 1.3 \cdot 10^{26} \text{ m}$$

Summary

- Evidence for conventional gravity is weak on large scales
- VSL is compatible with SR and GR
- It quantitatively implements Mach's principle
- It yields a theoretical foundation of the equivalence principle
- A dependence of c on the mass distribution is proposed that reproduces Newton's law
- Visible matter in this case suffices to resolve the flatness problem